A taste of statistics and applications to X-ray spectral fitting

✓ Normal error (Gaussian) distribution

most important in statistical analysis of data, describes the distribution of random observations for many experiments

Poisson distribution

 generally appropriate for counting experiments related to random processes (e.g., radioactive decay of elementary particles)

✓ Statistical tests: χ^2 and F-test

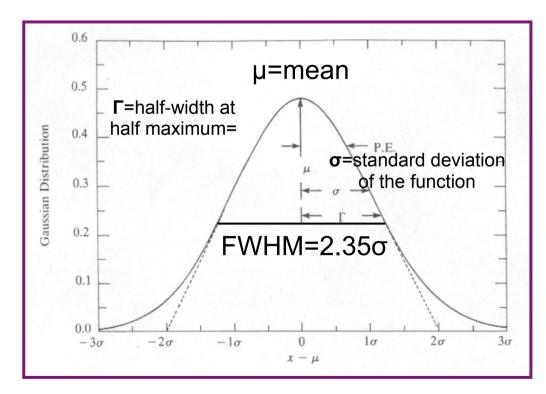
Further details in the XSPEC presentation

The Gaussian (normal error) distribution. I.

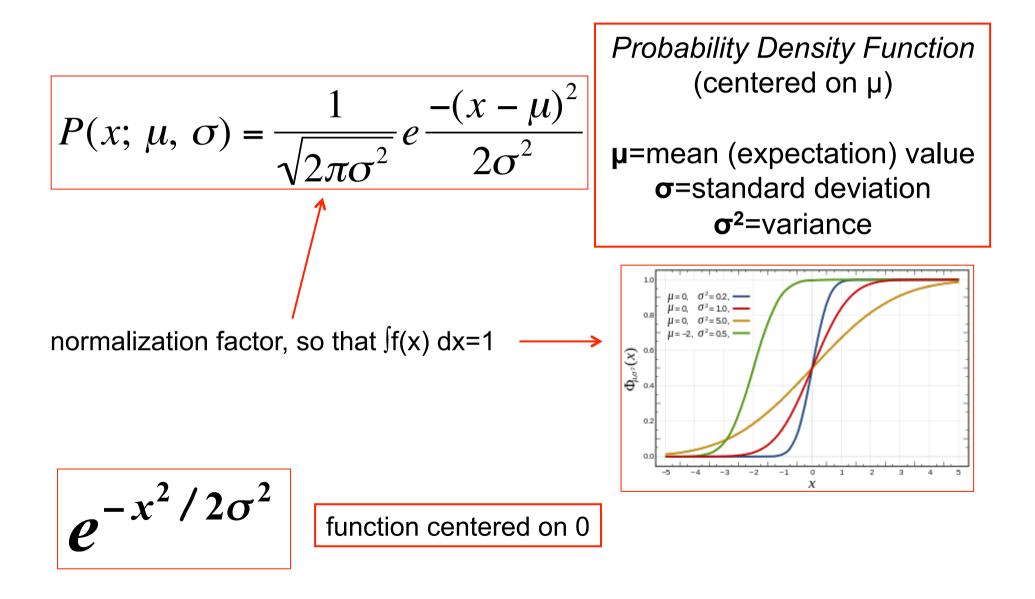
Averages of random variables (sufficiently large in number) independently drawn from independent distributions converge in distribution to the normal

Casual errors are above and below the "true" (most "common") value

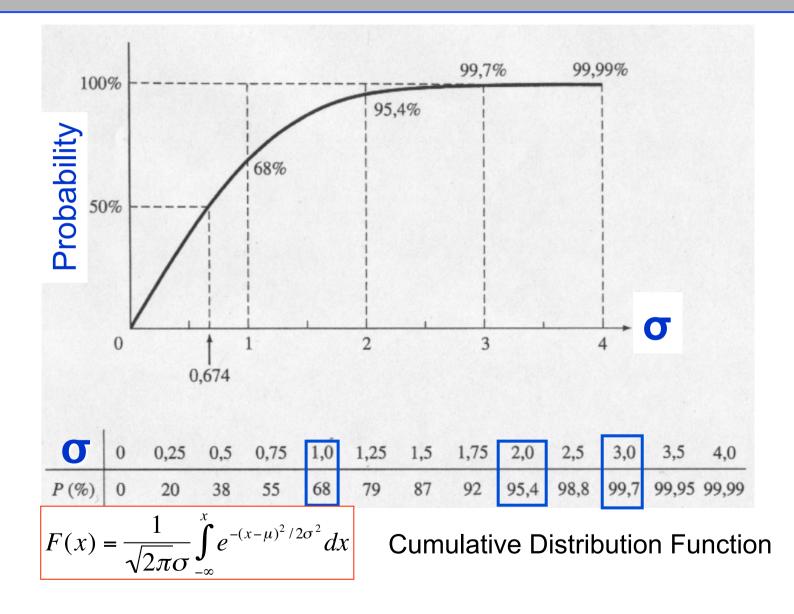
→ bell-shape distribution if systematic errors are negligible

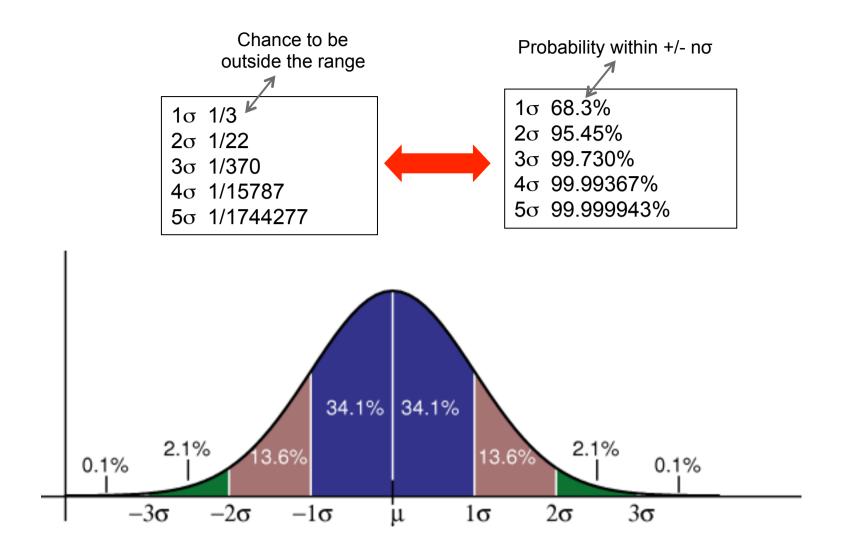


The Gaussian probability function. II.



The Gaussian probability function. III.





$$F(x) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{x} e^{-(x-\mu)^2/2\sigma^2} dx$$

Cumulative Distribution Function

The Poisson distribution

Describes experimental results where events are counted and the uncertainty is not related to the measurement but reflects the intrinsically casual behavior of the process (e.g., radioactive decay of particles, X-ray photons, etc.)

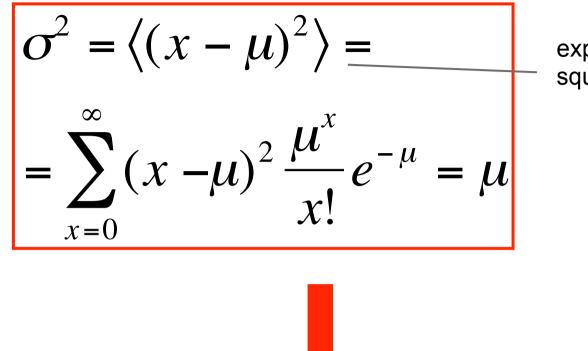
$$P(x) = e^{-\mu} \mu^{x} / x! \quad (x=0,1,2,...)$$

Probability of obtaining x events when µ events are expected x=observed number of events in a time interval (frequency of events)

average
number
of events
$$\overline{x} = \sum_{x=0}^{\infty} xP(x) = \sum_{x=0}^{\infty} xe^{-\mu} \mu^{x} / x! = \mu$$

averag numbe

 \rightarrow µ=average number of expected events if the experiment is repeated many times

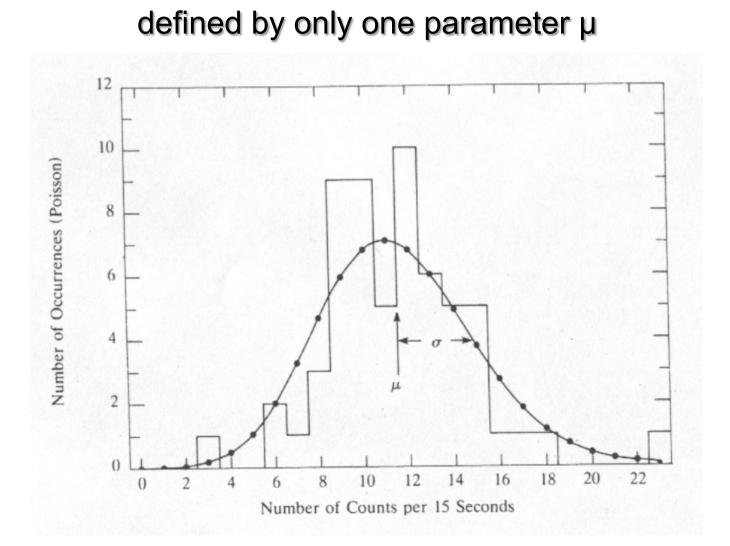


expectation value of the square of the deviations

the Poisson distribution with average counts= μ has standard deviation $\sqrt{\mu}$

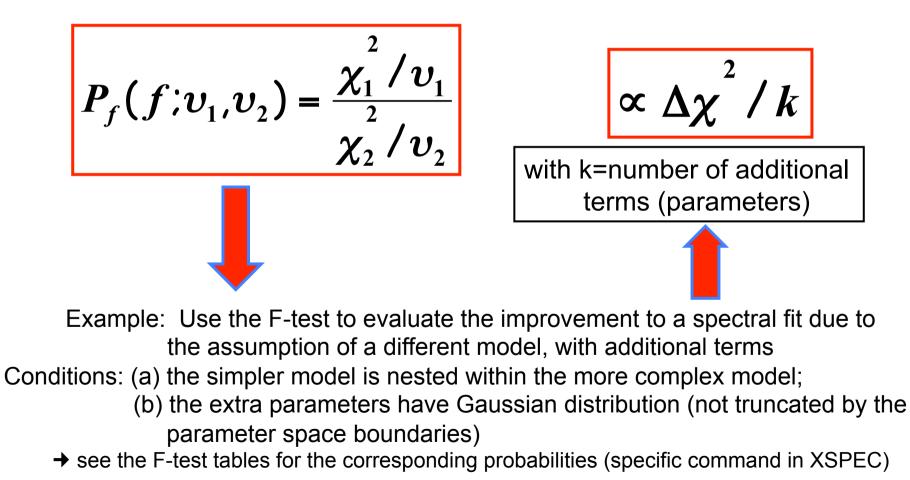
Example:
$$N_{counts} \pm \sqrt{N}$$

High µ: the Poisson distribution is approximated by the Gaussian distribution

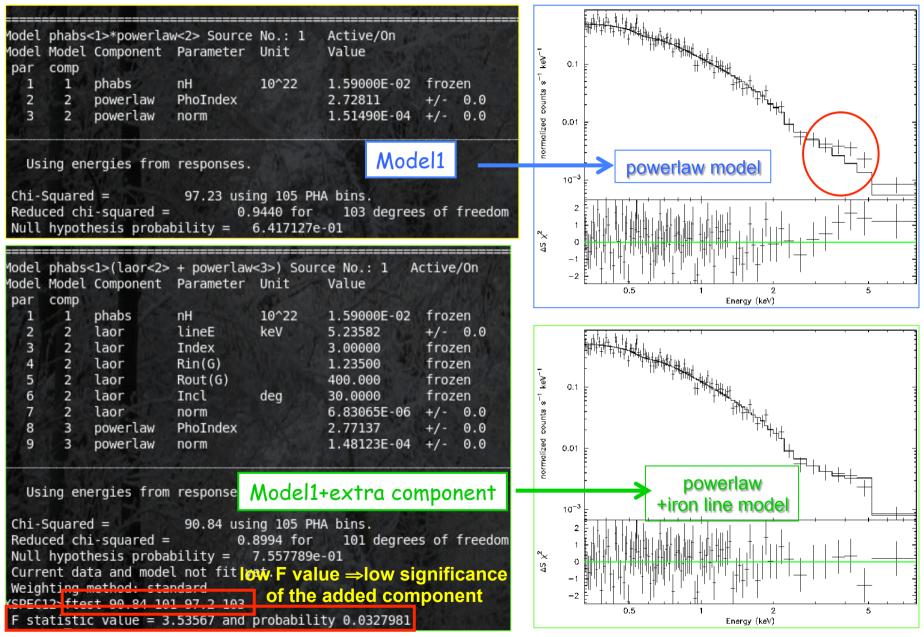


F-test

If two statistics following the χ^2 distribution have been determined, the ratio of the reduced chi-squares is distributed according to the F distribution



An application of the F-test within XSPEC



Fit (2) = Fit (1) + one component

xspec> ftest χ^2 (best fit) **dof** (best fit) χ^2 (previous fit) **dof** (previous fit)

xspec> ftest 90.8 101 97.2 103 → ftest=3.55 → prob=0.0328

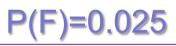
 $F_t = \left(\frac{\chi^2(dof) - \chi^2(dof - k)}{dof - (dof - k)}\right) / (\chi^2(dof - k)/(dof - k)) =$ = $(\Delta \chi^2 / k) / \chi_{\nu}^2$ Ex: $\chi^2(103) = 97.23$ $\chi^2(101) = 90.84$ $\rightarrow \Delta \chi^2 = 6.39, k = 2 \rightarrow F_t = (6.39/2)/(90.84/101) = 3.55$

 F_t follows the F distribution with $v_1 = k = \Delta(dof)$ and $v_2 = dof - k(-1)$

Search in the F-distribution tables for the probability of the null hypothesis (H₀) for v_1 =2 and v_2 =100

									TAD	LE 5	(Can											
v ₁ =2		_								P(F) =		.)									230	
	f2 11	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	8		
v ₂ =100	2345		$ \begin{array}{r} 19.00 \\ 9.55 \\ 6.94 \\ 5.79 \end{array} $		224.58 19.25 9.12 6.39 5.19	230.16 19.30 9.01 6.26 5.05					$ \begin{array}{r} 19.40 \\ 8.79 \\ 5.96 \\ 4.74 \end{array} $	$ \begin{array}{r} 19.41 \\ 8.74 \\ 5.91 \\ 4.68 \end{array} $		$19.45 \\ 8.66 \\ 5.80 \\ 4.56$	$ \begin{array}{r} 19.45 \\ 8.64 \\ 5.77 \\ 4.53 \end{array} $		$19.47 \\ 8.59 \\ 5.72 \\ 4.46$	$19.48 \\ 8.57 \\ 5.69 \\ 4.43$	$19.49 \\ 8.55 \\ 5.66 \\ 4.40$	$19.50 \\ 8.53 \\ 5.63 \\ 4.36$		
(range 60-120)	6 7 8 9 10		$5.14 \\ 4.74 \\ 4.46 \\ 4.26 \\ 4.10$	$\begin{array}{r} 4.76 \\ 4.35 \\ 4.07 \\ 3.86 \\ 3.71 \end{array}$	$\begin{array}{r} 4.53 \\ 4.12 \\ 3.84 \\ 3.63 \\ 3.48 \end{array}$	4.39 3.97 3.69 3.48 3.33	4.28 3.87 3.58 3.37 3.22	$\begin{array}{r} 4.21 \\ 3.79 \\ 3.50 \\ 3.29 \\ 3.14 \end{array}$	$\begin{array}{r} 4.15 \\ 3.73 \\ 3.44 \\ 3.23 \\ 3.07 \end{array}$	$\begin{array}{r} 4.10 \\ 3.68 \\ 3.39 \\ 3.18 \\ 3.02 \end{array}$	$\begin{array}{r} 4.06 \\ 3.64 \\ 3.35 \\ 3.14 \\ 2.98 \end{array}$	4.00 3.57 3.28 3.07 2.91	$3.94 \\ 3.51 \\ 3.22 \\ 3.01 \\ 2.84$	3.87 3.44 3.15 2.94 2.77	$3.84 \\ 3.41 \\ 3.12 \\ 2.90 \\ 2.74$	$3.81 \\ 3.38 \\ 3.08 \\ 2.86 \\ 2.70$	3.77 3.34 3.04 2.83 2.66	$3.74 \\ 3.30 \\ 3.01 \\ 2.79 \\ 2.62$	3.70 3.27 2.97 2.75 2.58	3.67 3.23 2.93 2.71 2.54		
	11 12 13 14 15		$3.98 \\ 3.89 \\ 3.81 \\ 3.74 \\ 3.68$	3.59 3.49 3.41 3.34 3.29	$3.36 \\ 3.26 \\ 3.18 \\ 3.11 \\ 3.06$	3.20 3.11 3.03 2.96 2.90	3.09 3.00 2.92 2.85 2.79	3.01 2.91 2.83 2.76 2.71	2.95 2.85 2.77 2.70 2.64	2.90 2.80 2.71 2.65 2.59	2.85 2.75 2.67 2.60 2.54	2.79 2.69 2.60 2.53 2.48	2.72 2.62 2.53 2.46 2.40	2.65 2.54 2.46 2.39 2.33	$2.61 \\ 2.51 \\ 2.42 \\ 2.35 \\ 2.29$	2.57 2.47 2.38 2.31 2.25	2.53 2.43 2.34 2.27 2.20	2.49 2.38 2.30 2.22 2.16	2.45 2.34 2.25 2.18 2.11	2.40 2.30 2.21 2.13 2.07		
F=3.15,3.07	16 17 18 19 20	$\begin{array}{r} 4.49 \\ 4.45 \\ 4.41 \\ 4.38 \\ 4.35 \end{array}$	3.63 3.59 3.55 3.52 3.49	$3.24 \\ 3.20 \\ 3.16 \\ 3.13 \\ 3.10$	3.01 2.96 2.93 2.90 2.87	2.85 2.81 2.77 2.74 2.71	2.74 2.70 2.66 2.63 2.60	2.66 2.61 2.58 2.54 2.51	2.59 2.55 2.51 2.48 2.45	2.54 2.49 2.46 2.42 2.39	2.49 2.45 2.41 2.38 2.35	2.42 2.38 2.34 2.31	2.35 2.31 2.27 2.23 2.23	2.28 2.23 2.19 2.16 2.12	2.24 2.19 2.15 2.11	2.19 2.15 2.11 2.07	2.15 2.10 2.06 2.03 1.00	2.11 2.06 2.02 1.98 1.95	2.06 2.01 1.97 1.93 1.90	2.01 1.96 1.92 1.88 1.84		
at P(F)=0.05	$ \begin{array}{r} 16\\17\\18\\19\\20\\\hline 21\\22\\23\\24\\25\\\hline \end{array} $		3.47 3.44 3.42 3.40 3.39	3.07 3.05 3.03 3.01	2.84 2.82 2.80 2.78 2.76	2.68 2.66 2.64 2.62 2.60	2.57 2.55 2.53 2.51 2.49	$2.49 \\ 2.46 \\ 2.44 \\ 2.42 \\ 2.40$	2.42 2.40 2.37 2.36 2.34	2.37 2.34 2.32 2.30 2.28	2.32 2.30 2.27 2.25	2.25 2.23 2.20 2.18	2.18 2.15 2.13 2.11	$2.12 \\ 2.07 \\ 2.05 \\ 2.03 \\ 2.01$	2.05 2.03 2.00 1.98	$2.01 \\ 1.98 \\ 1.96 \\ 1.94 \\ 1.92$	1.96 1.94 1.91 1.89 1.87	$ 1.92 \\ 1.89 \\ 1.86 \\ 1.84 \\ 1.82 $	1.87 1.84 1.81 1.79 1.77	1.81 1.78 1.76 1.73 1.71		
	23 27 28 29 30		3.37 3.35 3.34 3.33 3.32	2.99 2.98 2.96 2.95 2.93	2.74 2.73 2.71 2.70	2.50 2.59 2.57 2.56 2.55 2.53	2.49 2.47 2.46 2.45 2.43 2.42	2.39 2.37 2.36 2.35 2.33	2.34 2.32 2.31 2.29 2.28 2.27	2.28 2.27 2.25 2.24 2.22 2.21	2.24 2.22 2.20 2.19 2.18	$2.16 \\ 2.15 \\ 2.13 \\ 2.12 \\ 2.10$	2.09 2.07 2.06 2.04 2.03	$2.01 \\ 1.99 \\ 1.97 \\ 1.96 \\ 1.94 \\ 1.93$	1.96 1.95 1.93 1.91 1.90	$ 1.92 \\ 1.90 \\ 1.88 \\ 1.87 \\ 1.85 \\ 1.84 $	$ 1.87 \\ 1.85 \\ 1.84 \\ 1.82 \\ 1.81 \\ 1.79 $	$ 1.82 \\ 1.80 \\ 1.79 \\ 1.77 \\ 1.75 \\ 1.74 \\ $	$1.77 \\ 1.75 \\ 1.73 \\ 1.71 \\ 1.70 \\ 1.68 $	$1.69 \\ 1.67 \\ 1.65$		
	60 120	4.08 4.00 3.92	3.15 3.07	2.92 2.84 2.76 2.68	2.69 2.61 2.53 2.45	2.45 2.37 2.29	$2.34 \\ 2.25 \\ 2.18$	2.25 2.17 2.09	$2.18 \\ 2.10 \\ 2.02$	$2.12 \\ 2.04 \\ 1.96$	2.16 2.08 1.99 1.91	2.09 2.00 1.92 1.83	1.92 1.84 1.75	$1.84 \\ 1.75 \\ 1.66$	1.89 1.79 1.70 1.61	$1.74 \\ 1.65 \\ 1.55$	$1.69 \\ 1.59 \\ 1.50$	$1.74 \\ 1.64 \\ 1.53 \\ 1.43$	1.68 1.58 1.47 1.35	1.64 1.62 1.51 1.39 1.25		
	0	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00		
F = 0.00	and the second second																					
F _{xspec} =3.55			_						_													
Xspec Close									C	P(F) =	0.025]							100			Ρ
XSpec Close	f2 f1	<u> </u>	2	3	4	5	6	7	8	9	10	12	15	20	24 97.25 1	30	40	60 1009.8 1	120 014.0 1	∞ 018.3		Ρ
v ₁ =2	f1 f2 1 64 3 1 4 5	47.79 79 38.51 3 17.44 1 12.22 1	_	$\begin{array}{r} 64.16 \\ 39.16 \\ 15.44 \\ 9.98 \\ 7.76 \end{array}$	899.58 9 39.25 15.10 9.60 7.39	$\begin{array}{c} 21.85 \\ 39.30 \\ 14.88 \\ 9.36 \\ 7.15 \end{array}$	$\begin{array}{r} 37.11 \\ 39.33 \\ 14.74 \\ 9.20 \\ 6.98 \end{array}$	$\begin{array}{r} 48.22 \\ 39.36 \\ 14.62 \\ 9.07 \\ 6.85 \end{array}$	8 56.66 9 39.37 14.54 8.98 6.76	9 63.28 9 39.39 14.47 8.90 6.68	$10 \\ 68.63 \\ 9 \\ 39.40 \\ 14.42 \\ 8.84 \\ 6.62 \\ $	$\begin{array}{c} 076.71 \\ 39.42 \\ 14.34 \\ 8.75 \\ 6.52 \end{array}$	$\begin{array}{r} 84.87 \\ 39.43 \\ 14.25 \\ 8.66 \\ 6.43 \end{array}$	$\begin{array}{r} 93.10 \\ 39.45 \\ 14.17 \\ 8.56 \\ 6.33 \end{array}$	$\begin{array}{c} 97.25 \\ 39.46 \\ 14.12 \\ 8.51 \\ 6.28 \end{array}$	$\begin{array}{c} 1001.4 \\ 39.46 \\ 14.08 \\ 8.46 \\ 6.23 \end{array}$	$\begin{array}{c} .005.6 \\ 39.47 \\ 14.04 \\ 8.41 \\ 6.18 \end{array}$	$\begin{array}{c} 009.8 \\ 39.48 \\ 13.99 \\ 8.36 \\ 6.12 \end{array}$	$\begin{array}{c} 014.0 \ 1\\ 39.49\\ 13.95\\ 8.31\\ 6.07 \end{array}$	018.3 39.50 13.90 8.26 6.02		Ρ
v ₁ =2	2 3 3 1 4 1 5 1 67 8 9	47.79 79 38.51 3 17.44 1 12.22 1 10.01	99.50 86 89.00 3 16.04 1 10.65 8.43	$\begin{array}{r} 64.16 \\ 39.16 \\ 15.44 \\ 9.98 \\ 7.76 \end{array}$	99.58 9 39.25	21.85 9 39.30	37.11 9 39.33 14.74 9.20	$\begin{array}{r} 48.22 & 9\\ 39.36 \\ 14.62 \\ 9.07 \\ 6.85 \\ 5.70 \\ 4.99 \\ 4.53 \\ 4.20 \\ 3.95 \end{array}$	8 56.66 9 39.37 14.54 8.98 6.76 5.60 4.90 4.43 4.10 3.85	9 63.28 9 39.39 14.47 8.90 6.68 5.52 4.82 4.82 4.36 4.03 3.78	$\begin{array}{c} 10 \\ 68.63 & 9 \\ 39.40 \\ 14.42 \\ 8.84 \\ 6.62 \\ 5.46 \\ 4.76 \\ 4.30 \\ 3.96 \\ 3.72 \end{array}$	$\begin{array}{c} 76.71 \\ 39.42 \\ 14.34 \\ 8.75 \\ 6.52 \\ 5.37 \\ 4.67 \\ 4.20 \\ 3.87 \\ 3.62 \end{array}$	$\begin{array}{r} 84.87 \\ 9\\ 39.43 \\ 14.25 \\ 8.66 \\ 6.43 \\ 5.27 \\ 4.57 \\ 4.10 \\ 3.77 \\ 3.52 \end{array}$	$\begin{array}{r} 93.10 \\ 93.45 \\ 14.17 \\ 8.56 \\ 6.33 \\ 5.17 \\ 4.47 \\ 4.00 \\ 3.67 \\ 3.42 \end{array}$	$\begin{array}{c} 97.25 \\ 39.46 \\ 14.12 \\ 8.51 \\ 6.28 \\ 5.12 \\ 4.42 \\ 3.95 \\ 3.61 \\ 3.37 \end{array}$	$\begin{array}{c} 1001.4 \\ 39.46 \\ 14.08 \\ 8.46 \\ 6.23 \\ 5.07 \\ 4.36 \\ 3.89 \\ 3.56 \\ 3.31 \end{array}$	$\begin{array}{c} 005.6 \\ 39.47 \\ 14.04 \\ 8.41 \\ 6.18 \\ 5.01 \\ 4.31 \\ 3.84 \\ 3.51 \\ 3.26 \end{array}$	$\begin{array}{c} 009.8 \\ 39.48 \\ 13.99 \\ 8.36 \\ 6.12 \\ 4.96 \\ 4.25 \\ 3.78 \\ 3.45 \\ 3.20 \end{array}$	$\begin{array}{c} 014.0 \ 1\\ 39.49\\ 13.95\\ 8.31\\ 6.07\\ 4.90\\ 4.20\\ 3.73\\ 3.39\\ 3.14 \end{array}$	$\begin{array}{c} 018.3\\ 39.50\\ 13.90\\ 8.26\\ 6.02\\ 4.85\\ 4.14\\ 3.67\\ 3.33\\ 3.08\\ \end{array}$		P
v ₁ =2 v ₂ =100	2 3 3 1 5 1 6 7 8 9 10 11 12 13	$\begin{array}{c} 17.79 & 79\\ 38.51 & 3\\ 17.44 & 1\\ 12.22 & 1\\ 10.01 \\ \hline 8.81 \\ 8.07 \\ 7.57 \\ 7.21 \\ 6.94 \\ \hline 6.72 \\ 6.55 \\ 6.41 \\ 6.30 \\ \hline \end{array}$	99.50 86 99.00 3 16.04 1 10.65 8.43 7.26 6.54 6.06 5.71 5.46	$\begin{array}{c} 64.16 \\ 839.16 \\ 15.44 \\ 9.98 \\ 7.76 \\ 6.60 \\ 5.89 \\ 5.42 \\ 5.08 \\ 4.83 \end{array}$	$\begin{array}{c} 399.58 & 9\\ 39.25 \\ 15.10 \\ 9.60 \\ 7.39 \\ 6.23 \\ 5.52 \\ 5.05 \\ 4.72 \\ 4.47 \end{array}$	$\begin{array}{c} 21.85 \\ 39.30 \\ 14.88 \\ 9.36 \\ 7.15 \\ 5.99 \\ 5.29 \\ 4.82 \\ 4.48 \\ 4.24 \end{array}$	$\begin{array}{r} 37.11 & 9\\ 39.33 \\ 14.74 \\ 9.20 \\ 6.98 \\ 5.82 \\ 5.12 \\ 4.65 \\ 4.32 \\ 4.07 \end{array}$	$\begin{array}{r} 48.22 \\ 39.36 \\ 14.62 \\ 9.07 \\ 6.85 \end{array}$	8 56.66 9 39.37 14.54 8.98 6.76	9 63.28 9 39.39 14.47 8.90 6.68	$10 \\ 68.63 \\ 9 \\ 39.40 \\ 14.42 \\ 8.84 \\ 6.62 \\ $	$\begin{array}{c} 0.76.71 & 9\\ 39.42 \\ 14.34 \\ 8.75 \\ 6.52 \\ 5.37 \\ 4.67 \\ 4.20 \\ 3.87 \end{array}$	$\begin{array}{r} 84.87 & 9\\ 39.43 \\ 14.25 \\ 8.66 \\ 6.43 \\ 5.27 \\ 4.57 \\ 4.57 \\ 3.77 \\ 3.52 \\ \hline 3.33 \\ 3.18 \\ 3.05 \\ 2.95 \\ 2.86 \\ \end{array}$	$\begin{array}{c} 93.10 \\ 93.45 \\ 39.45 \\ 8.56 \\ 6.33 \\ 5.17 \\ 4.47 \\ 4.07 \\ 3.67 \\ 3.42 \\ \hline 3.23 \\ 3.07 \\ 2.95 \\ 2.84 \\ 2.76 \end{array}$	$\begin{array}{c} 97.25 \\ 39.46 \\ 14.12 \\ 8.51 \\ 6.28 \\ 5.12 \\ 4.42 \\ 3.95 \\ 3.61 \\ 3.37 \\ 3.17 \\ 3.02 \\ 2.89 \\ 2.79 \\ 2.70 \end{array}$	$\begin{array}{c} 0001.4 \\ 1\\ 39.46 \\ 8.46 \\ 6.23 \\ 5.07 \\ 4.36 \\ 3.89 \\ 3.56 \\ 3.31 \\ \hline 3.12 \\ 2.96 \\ 2.84 \\ 2.73 \\ 2.64 \end{array}$	$\begin{array}{c} 005.6 & 1\\ 39.47 \\ 14.04 \\ 8.41 \\ 6.18 \\ 5.01 \\ 4.31 \\ 3.51 \\ 3.26 \\ \hline 3.26 \\ 3.06 \\ 2.91 \\ 2.78 \\ 2.67 \\ 2.58 \end{array}$	$\begin{array}{c} 009.8 & 1\\ 39.48 \\ 13.99\\ 8.36\\ 6.12\\ 4.96\\ 4.25\\ 3.78\\ 3.45\\ 3.20\\ \hline 3.00\\ 2.85\\ 2.72\\ 2.61\\ 2.52\\ \end{array}$	$\begin{array}{c} 014.0 \ 1\\ 39.49\\ 13.95\\ 8.31\\ 6.07\\ 4.90\\ 4.20\\ 3.73\\ 3.39\\ 3.14\\ 2.94\\ 2.79\\ 2.66\\ 2.55\\ 2.46\\ \end{array}$	$\begin{array}{c} 018.3\\ 39.50\\ 13.90\\ 8.26\\ 6.02\\ \hline 4.85\\ 4.14\\ 3.67\\ 3.33\\ 3.08\\ 2.88\\ 2.72\\ 2.60\\ 2.49\\ 2.40\\ \end{array}$		P
v ₁ =2	2 3 3 4 5 1 6 7 8 9 10 11 12 13 14 15	$\begin{array}{c} 17.79 & 79 \\ 88.51 & 31 \\ 17.44 & 1 \\ 12.22 & 1 \\ 10.01 \\ 8.81 \\ 8.07 \\ 7.57 \\ 7.21 \\ 6.94 \\ 6.72 \\ 6.52 \\ 6.41 \\ 6.30 \\ 6.20 \end{array}$	$\begin{array}{c} 99.50 & 86\\ 99.00 & 3\\ 0.654 & 6.064\\ 6.065 & 8.43\\ 7.26 & 6.564\\ 6.06 & 5.71 & 5.46\\ 5.71 & 5.46 & 5.10\\ 5.26 & 5.10 & 4.97\\ 4.86 & 4.76 & \end{array}$	$\begin{array}{c} 64.16 & 8\\ 39.16 \\ 9.98 \\ 7.76 \\ 6.60 \\ 5.89 \\ 5.42 \\ 5.08 \\ 4.83 \\ \hline 4.63 \\ 4.47 \\ 4.35 \\ 4.24 \\ 4.15 \\ \end{array}$	$\begin{array}{c} & & & & & \\ & & & \\ & & &$	$\begin{array}{c} 21.85 \\ 9.39.30 \\ 14.88 \\ 9.36 \\ 7.15 \\ 5.99 \\ 5.29 \\ 4.82 \\ 4.48 \\ 4.24 \\ \hline 4.04 \\ 3.89 \\ 3.77 \\ 3.66 \\ 3.58 \end{array}$	$\begin{array}{r} 37.11 \\ 39.33 \\ 14.74 \\ 9.20 \\ 6.98 \end{array}$	$\begin{array}{r} 48.22 & 9\\ 39.36 & \\ 14.62 & \\ 9.07 & \\ 6.85 & \\ 5.70 & \\ 4.99 & \\ 4.53 & \\ 4.20 & \\ 3.95 & \\ 3.95 & \\ \hline 3.76 & \\ 3.61 & \\ 3.48 & \\ 3.38 & \\ 3.29 & \\ \end{array}$	8 56.66 9 39.37 14.54 8.98 6.76 5.60 4.90 4.43 4.10 3.85 3.66 3.51 3.39 3.29 3.20	9 63.28 9 39.39 14.47 8.90 6.68 5.52 4.82 4.36 4.03 3.78 3.78 3.59 3.44 3.31 3.21 3.12	$\begin{array}{c} 10 \\ 68.63 & 9 \\ 39.40 \\ 14.42 \\ 8.84 \\ 6.62 \\ 5.46 \\ 4.76 \\ 4.30 \\ 3.96 \\ 3.72 \end{array}$	$\begin{array}{c} 076.71 \\ 39.42 \\ 14.34 \\ 8.75 \\ 6.52 \\ 5.37 \\ 4.67 \\ 4.20 \\ 3.87 \\ 3.62 \\ \hline 3.43 \\ 3.28 \\ 3.15 \\ 3.05 \end{array}$	$\begin{array}{r} 84.87 & 9\\ 39.43 \\ 14.25 \\ 8.66 \\ 6.43 \\ 5.27 \\ 4.57 \\ 4.57 \\ 3.77 \\ 3.52 \\ \hline 3.33 \\ 3.18 \\ 3.05 \\ 2.95 \\ 2.86 \\ \end{array}$	$\begin{array}{r} 93.10 \\ 93.45 \\ 14.17 \\ 8.56 \\ 6.33 \\ 5.17 \\ 4.47 \\ 4.00 \\ 3.67 \\ 3.42 \end{array}$	$\begin{array}{c} 97.25 \\ 39.46 \\ 14.12 \\ 8.51 \\ 6.28 \\ 5.12 \\ 4.42 \\ 3.95 \\ 3.61 \\ 3.37 \\ 3.17 \\ 3.02 \\ 2.89 \\ 2.79 \\ 2.70 \\ 2.63 \\ 2.56 \\ 2.56 \\ 2.56 \\ 2.41 \end{array}$	$\begin{array}{c} 001.4 \ 1\\ 39.46\\ 14.08\\ 8.46\\ 6.23\\ 5.07\\ 4.36\\ 3.89\\ 3.56\\ 3.31\\ 3.12\\ 2.984\\ 2.73\\ 2.64\\ 2.57\\ 2.50\\ 2.44\\ 2.39\\ 2.35\\ \end{array}$	$\begin{array}{c} 005.6 \\ 39.47 \\ 14.04 \\ 8.41 \\ 6.18 \\ 5.01 \\ 4.31 \\ 3.51 \\ 3.26 \\ 3.06 \\ 2.91 \\ 2.67 \\ 2.58 \\ 2.67 \\ 2.51 \\ 2.44 \\ 2.38 \\ 2.38 \\ 2.29 \end{array}$	$\begin{array}{c} 009.8 & 1\\ 39.48\\ 39.48\\ 6.12\\ 4.96\\ 4.25\\ 3.78\\ 3.45\\ 3.20\\ 3.60\\ 2.85\\ 2.72\\ 2.61\\ 2.52\\ 2.45\\ 2.38\\ 2.32\\ 2.22\\ 2.22\\ \end{array}$	$\begin{array}{c} 014.0 \ 1\\ 39.49\\ 39.49\\ 6.07\\ 4.20\\ 3.73\\ 3.39\\ 3.14\\ 2.94\\ 2.76\\ 2.55\\ 2.46\\ 2.55\\ 2.46\\ 2.38\\ 2.32\\ 2.26$	$\begin{array}{c} 018.3\\ 39.50\\ 13.90\\ 8.26\\ 6.02\\ 4.85\\ 4.14\\ 3.67\\ 3.33\\ 3.08\\ 2.82\\ 2.60\\ 2.49\\ 2.49\\ 2.49\\ 2.49\\ 2.49\\ 2.49\\ 2.49\\ 2.25\\ 2.19\\ 2.13\\ 2.09\\ \end{array}$		P
v ₁ =2 v ₂ =100 (range 60-120)	2 3 4 3 4 5 7 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	$\begin{array}{c} 17.79 & 79 \\ 88.51 & 38.51 \\ 88.51 & 30 \\ 10.01 \\ 8.81 \\ 8.07 \\ 7.57 \\ 7.21 \\ 6.94 \\ 6.72 \\ 6.54 \\ 6.41 \\ 6.30 \\ 6.20 \\ 6.12 \\ 6.04 \\ 5.98 \\ 5.92 \\ 5.87 \\ \end{array}$	$\begin{array}{c} 99.50 & 86\\ 99.00 & 3\\ 6.04 & 3\\ 8.43 & \\ 8.44 & \\$	$\begin{array}{c} 64.16 & 8\\ 39.16 & \\ 15.44 & \\ 9.98 & \\ 7.76 & \\ 6.60 & \\ 5.89 & \\ 5.42 & \\ 5.08 & \\ 4.83 & \\ 4.47 & \\ 4.83 & \\ 4.63 & \\ 4.47 & \\ 4.24 & \\ 4.15 & \\ 4.08 & \\ 4.01 & \\ 3.95 & \\ 3.90 & \\ 3.86 & \\ \end{array}$	$\begin{array}{c} & & & & & \\ & & & \\ & & & & \\$	$\begin{array}{c} 21.85 & 9\\ 39.30\\ 14.88\\ 9.36\\ 7.15\\ 7.15\\ 7.15\\ 5.99\\ 4.82\\ 4.48\\ 4.24\\ 4.24\\ 4.04\\ 3.89\\ 3.77\\ 3.66\\ 3.56\\ 3.50\\ 3.44\\ 3.38\\ 3.32\\ 9\end{array}$	$\begin{array}{c} 37.11 & 9\\ 39.33 \\ 14.74 \\ 9.20 \\ 6.98 \\ 5.82 \\ 5.12 \\ 4.65 \\ 4.32 \\ 4.07 \\ 3.88 \\ 3.73 \\ 3.60 \\ 3.50 \\ 3.50 \\ 3.51 \\ 3.34 \\ 3.28 \\ 3.22 \\ 3.17 \\ 3.13 \end{array}$	$\begin{array}{r} 48.22 & 9\\ 39.36 \\ 14.62 \\ 9.07 \\ 6.85 \\ \hline \\ 5.70 \\ 4.99 \\ 4.53 \\ 4.20 \\ 3.95 \\ \hline \\ 3.95 \\ \hline \\ 3.76 \\ 3.61 \\ 3.48 \\ 3.28 \\ 3.29 \\ \hline \\ 3.216 \\ 3.10 \\ 3.05 \\ \hline \\ 3.01 \\ \end{array}$	8 56.66 9 39.37 14.54 8.98 6.76 5.60 4.43 4.43 4.10 5.60 4.43 4.10 3.66 3.51 3.29 3.20 3.12 3.01 2.96 2.91	9 63.28 9 39.39 14.47 8.90 6.68 5.52 4.82 4.82 4.36 4.03 3.78	$\begin{array}{c} 10\\ 68.63 & 9\\ 39.40\\ 14.42\\ 8.84\\ 6.62\\ 5.46\\ 4.76\\ 4.30\\ 3.96\\ 3.72\\ \hline 3.53\\ 3.37\\ 3.25\\ 3.15\\ 3.06\\ \end{array}$	$\begin{array}{c} 76.71 & 9\\ 39.42 \\ 39.42 \\ 8.75 \\ 6.52 \\ 5.37 \\ 4.67 \\ 4.20 \\ 3.87 \\ 3.62 \\ 3.43 \\ 3.28 \\ 3.15 \\ 3.05 \\ 2.96 \\ 2.89 \\ 2.82 \\ 2.77 \\ 2.72 \\ 2.77 \end{array}$	84.87 9 39.43 14.25 8.66 6.43 5.27 4.57 4.10 3.77 3.52 3.33 3.18 3.05 2.95	$\begin{array}{c} 93.10 \\ 93.45 \\ 39.45 \\ 8.56 \\ 6.33 \\ 5.17 \\ 4.47 \\ 4.07 \\ 3.67 \\ 3.42 \\ \hline 3.23 \\ 3.07 \\ 2.95 \\ 2.84 \\ 2.76 \end{array}$	$\begin{array}{c} 97.25 \\ 39.46 \\ 14.12 \\ 8.51 \\ 6.28 \\ 5.12 \\ 4.42 \\ 3.95 \\ 3.61 \\ 3.37 \\ 3.17 \\ 3.02 \\ 2.89 \\ 2.79 \\ 2.70 \end{array}$	$\begin{array}{c} 10011.4 \ 1\\ 39.46\\ 14.08\\ 8.46\\ 6.23\\ 5.07\\ 4.36\\ 3.89\\ 3.56\\ 3.31\\ \hline 3.31\\ \hline 3.12\\ 2.984\\ 2.73\\ 2.64\\ 2.73\\ 2.50\\ 2.44\\ 2.39\\ 2.35\\ \hline 2.31\\ 2.24\\ 2.21\\ 2.21\\ 2.21\\ 2.21\\ 2.18\\ \end{array}$	$\begin{array}{c} 0.005.6 & 1\\ 39.47 \\ 14.04 \\ 8.41 \\ 6.18 \\ 5.01 \\ 4.31 \\ 3.26 \\ 3.26 \\ 3.26 \\ 2.51 \\ 2.78 \\ 2.51 \\ 2.44 \\ 2.33 \\ 2.29 \\ 2.25 \\ 2.21 \\ 2.18 \\ 2.12 \\ \end{array}$	$\begin{array}{c} 009.8 \ 1\\ 39.48\\ 13.99\\ 8.36\\ 6.12\\ 4.96\\ 3.78\\ 3.45\\ 3.78\\ 3.45\\ 3.78\\ 3.45\\ 3.20\\ 2.61\\ 2.52\\ 2.61\\ 2.52\\ 2.61\\ 2.52\\ 2.45\\ 2.38\\ 2.32\\ 2.21\\ 2.18\\ 2.22\\ 2.21\\ 2.18\\ 2.05\\ \end{array}$	$\begin{array}{c} 014.0 \ 1\\ 39.49 \\ 13.95 \\ 8.31 \\ 6.07 \\ 4.90 \\ 4.20 \\ 3.73 \\ 3.39 \\ 3.14 \\ 2.94 \\ 2.55 \\ 2.46 \\ 2.38 \\ 2.32 \\ 2.20 \\ 2.16 \\ 2.11 \\ 2.08 \\ 2.04 \\ 2.01 \\ 1.98 \\ \end{array}$	$\begin{array}{c} 018.3\\ 39.50\\ 13.90\\ 8.26\\ 6.02\\ 4.85\\ 4.14\\ 3.67\\ 3.308\\ 2.72\\ 2.30\\ 2.49\\ 2.40\\ 2.32\\ 2.25\\ 2.19\\ 2.25\\ 2.19\\ 2.09\\ 2.04\\ 2.00\\ 1.97\\ 1.91\\ 1.91\\ \end{array}$		P
v ₁ =2 v ₂ =100 (range 60-120) F=3.93,3.80	2 2 3 3 1 1 5 1 6 7 7 8 8 8 9 9 10 11 12 13 13 14 15 16 16 17 7 18 9 9 9 9 0 0 21 22 22 22 22 22 22 22 22 22 22 22 22	$\begin{array}{c} 47.79 & 79 \\ 88.51 & 37 \\ 88.51 & 37 \\ 12.22 & 1 \\ 10.01 & 8.81 \\ 8.07 & 7.51 \\ 6.94 & 6.72 \\ 6.75 & 6.41 \\ 6.55 & 6.41 \\ 6.55 & 6.41 \\ 6.630 & 6.20 \\ 6.12 & 6.04 \\ 5.98 & 5.92 \\ 5.87 & 5.87 \\ 5.83 \\ 5.79 & 5.87 \\ 5.69 \end{array}$	$\begin{array}{c} 99.50 & 86\\ 99.00 & 5\\ 60.04 & 5\\ 8.43 & 7.26 & 6.546 & 5\\ 5.46 & 5.76 & 5.46 & 5\\ 5.26 & 5.16 & 5.46 & 5\\ 5.26 & 5.10 & 4.97 & 4.86 & 4.51 & 4.46 & 4.51 & 4.46 & 4.51 & 4.46 & 4.51 & 4.46 & 4.51 & 4.46 & 4.35 & 4.32 & 4.29 & $	$\begin{array}{c} 64.16 & 8\\ 39.16 \\ 15.44 \\ 9.98 \\ 7.76 \\ 6.60 \\ 5.89 \\ 5.42 \\ 5.08 \\ 4.83 \\ 4.47 \\ 4.35 \\ 4.24 \\ 4.15 \\ 4.08 \\ 4.01 \\ 3.95 \\ 3.90 \\ 3.86 \\ 3.86 \\ 3.75 \\ 3.75 \\ 3.75 \\ 3.75 \\ 3.69 \end{array}$	$\begin{array}{c} & & & & & \\ & & & \\ &$	$\begin{array}{c} 21.85 & 9\\ 39.30\\ 14.88\\ 9.36\\ 7.15\\ 5.99\\ 5.29\\ 4.82\\ 4.24\\ 4.24\\ 4.24\\ 4.24\\ 4.24\\ 3.87\\ 3.66\\ 3.58\\ 3.58\\ 3.50\\ 3.44\\ 3.38\\ 3.29\\ 3.25\\ 3.22\\ 3.18\\ 3.13\\ \end{array}$	$\begin{array}{c} 37.11 & 9\\ 39.33 \\ 99.20 \\ 6.98 \\ 5.82 \\ 4.65 \\ 5.12 \\ 4.65 \\ 4.32 \\ 4.07 \\ 3.88 \\ 3.73 \\ 3.60 \\ 3.50 \\ 3.41 \\ 3.34 \\ 3.28 \\ 3.22 \\ 3.17 \\ 3.13 \\ 3.09 \\ 3.09 \\ 3.02 \\ 2.99 \\ 2.97 \\ 2.94 \\ 2.92 \\ 2.90 \\ 2.88 \end{array}$	$\begin{array}{r} 48.22 & 9\\ 39.36 & \\ 14.62 & \\ 9.07 & \\ 6.85 & \\ 5.70 & \\ 4.99 & \\ 4.53 & \\ 4.20 & \\ 3.95 & \\ 3.95 & \\ \hline 3.76 & \\ 3.61 & \\ 3.48 & \\ 3.38 & \\ 3.29 & \\ \end{array}$	8 56.66 9 39.37 4.8.98 6.76 5.60 4.40 3.85 3.60 3.51 3.39 3.20 3.12 3.01 2.91 2.87 2.81 2.75	9 63.28 9 39.39 39 39.39 44.47 8.90 6 6.68 5.52 4.36 4.36 4.36 3.78 3.78 3.78 3.59 3.41 3.31 3.21 3.321 3.41 3.31 2.98 2.98 2.884 2.884 2.80 2.73 2.68	10 668.63 9 339.40 6.62 5.46 6.62 5.46 6.62 5.46 4.30 3.37 3.37 3.37 3.37 3.37 3.37 3.37 3.37 2.87 2.87 2.87 2.87 2.64 2.54 3.56 2.57 3.70 2.67 2.57 3.57 2.55 3.57 2.55 3.57 2.55 3.57 2.55 3.57 2.55 3.57 2.55 3.57 2.	$\begin{array}{c} 776.71 & 9\\ 39.42 \\ 14.34 \\ 8.75 \\ 6.52 \\ 5.37 \\ 4.67 \\ 4.20 \\ 3.87 \\ 3.62 \\ 3.43 \\ 3.28 \\ 3.15 \\ 3.05 \\ 2.96 \\ 2.89 \\ 2.82 \\ 2.77 \\ 2.68 \end{array}$	$\begin{array}{c} 84.87 & 9\\ 39.43 \\ 14.25 \\ 8.66 \\ 6.43 \\ 5.27 \\ 4.57 \\ 4.57 \\ 3.77 \\ 3.52 \\ 3.33 \\ 3.18 \\ 3.05 \\ 2.95 \\ 2.86 \\ 2.79 \\ 2.72 \\ 2.67 \\ 2.62 \\ 2.57 \end{array}$	$\begin{array}{c} 93.10 \\ 93.45 \\ 39.45 \\ 14.17 \\ 8.56 \\ 6.33 \\ 5.17 \\ 4.47 \\ 4.00 \\ 3.67 \\ 3.42 \\ \hline 3.23 \\ 3.07 \\ 2.95 \\ 2.84 \\ 2.76 \\ 2.84 \\ 2.76 \\ 2.84 \\ 2.68 \\ 2.51 \\ 2.46 \\ \hline 2.42 \\ 2.36 \\ 2.36 \\ 2.30 \\ \hline 2.30 \end{array}$	$\begin{array}{c} 97.25 \\ 39.46 \\ 14.12 \\ 8.51 \\ 6.28 \\ 5.12 \\ 4.42 \\ 3.95 \\ 3.61 \\ 3.37 \\ 3.02 \\ 2.89 \\ 2.79 \\ 2.79 \\ 2.63 \\ 2.56 \\ 2.56 \\ 2.41 \\ 2.37 \\ 2.30 \\ 2.30 \\ 2.24 \\ \end{array}$	$\begin{array}{c} 0.001.4 & 1\\ 3.9.46\\ 14.08\\ 8.46\\ 6.23\\ 5.07\\ 4.36\\ 3.89\\ 3.56\\ 3.31\\ 3.31\\ 3.12\\ 2.984\\ 2.73\\ 2.64\\ 2.73\\ 2.57\\ 2.50\\ 2.44\\ 2.39\\ 2.35\\ 2.35\\ 2.35\\ 2.21\\ 2.22\\ $	$\begin{array}{c} 005.6 \\ 39.47 \\ 14.04 \\ 8.41 \\ 6.18 \\ 5.01 \\ 4.31 \\ 3.51 \\ 3.26 \\ 3.06 \\ 2.91 \\ 2.67 \\ 2.58 \\ 2.67 \\ 2.51 \\ 2.44 \\ 2.38 \\ 2.38 \\ 2.29 \end{array}$	$\begin{array}{c} 009.8 & 1\\ 39.48 \\ 13.99 \\ 8.36 \\ 6.12 \\ 4.96 \\ 4.25 \\ 3.78 \\ 3.45 \\ 3.78 \\ 3.45 \\ 3.20 \\ 2.85 \\ 2.761 \\ 2.52 \\ 2.61 \\ 2.52 \\ 2.45 \\ 2.32 \\ 2.22 \\ 2.18 \\ 2.14 \\ 2.11 \\ 2.11 \end{array}$	$\begin{array}{c} 014.0 \ 1\\ 39.49\\ 39.49\\ 6.07\\ 4.20\\ 3.73\\ 3.39\\ 3.14\\ 2.94\\ 2.76\\ 2.55\\ 2.46\\ 2.55\\ 2.46\\ 2.38\\ 2.32\\ 2.26$	018.3 039.50 018.3 039.50 6.02 4.85 4.14 3.67 2.88 2.43 2.72 2.49 2.25 2.13 2.09 2.25 2.13 2.09 2.249 2.25 2.13 2.09 2.13 2.09 2.13 2.04 1.91 1.91 1.91 1.91 1.91 1.91 1.91 1.9		P
v ₁ =2 v ₂ =100 (range 60-120)	2 1 3 1 4 1 5 6 7 8 9 9 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 24 24 26 26	$\begin{array}{c} 17.79 & 79 \\ 88.51 & 38.51 \\ 88.51 & 30 \\ 10.01 \\ 8.81 \\ 8.07 \\ 7.57 \\ 7.21 \\ 6.94 \\ 6.72 \\ 6.54 \\ 6.41 \\ 6.30 \\ 6.20 \\ 6.12 \\ 6.04 \\ 5.98 \\ 5.92 \\ 5.87 \\ \end{array}$	$\begin{array}{c} 99.50 & 86\\ 99.00 & 3\\ 6.04 & 3\\ 8.43 & \\ 8.44 & \\$	4.16 8 939.16 939.16 539.16 5.89 9.98 6.60 5.89 5.08 4.83 5.08 4.63 4.43 4.43 4.43 4.43 4.43 4.43 4.43 4.43 4.43 4.43 4.43 4.43 4.43 3.95 3.95 3.96 3.86 3.82 3.76 3.69 3.66 3.82 3.76 3.69 3.66 3.82 3.76 3.69 3.66 3.82 3.76 3.69 3.66 3.82 3.76 3.69 3.66 3.82 3.76 3.69 3.66 3.82 3.76 3.69 3.66 3.82 3.76 3.69 3.66 3.82 3.76 3.69 3.66 3.82 3.69 3.66 3.82 3.69 3.66 3.66 3.82 3.69 3.66 3.66 3.82 3.69 3.66 3.66 3.66 3.82 3.69 3.66 3.66 3.66 3.66 3.82 3.69 3.66 3.66 3.66 3.66 3.66 3.66 3.66 3.82 3.69 3.66	$\begin{array}{c} 99,58 \\ 39,25 \\ 39,25 \\ 15,10 \\ 15,10 \\ 7,39 \\ 6,23 \\ 5,52 \\ 4,72 \\ 4,28 \\ 4,47 \\ 4,28 \\ 4,47 \\ 4,44 \\ 4,44 \\ 4,42 \\ 4,44 \\ 4,42 \\ 4,44 \\ 4,44 \\ 4,42 \\ 4,44 \\ 4,33 \\ 8,38 \\ 3,36 \\ 3,36 \\ 3,36 \\ 3,36 \\ 3,36 \\ 3,36 \\ 3,36 \\ 3,36 \\ 3,36 \\ 3,36 \\ 3,36 \\ 3,36 \\ 3,37 \\ 3,37 \\ 3,31 \\ 3,31 \\ 3,29 \\ 3,27 \\ 3,27 \\ 3,33 \\ 3,31 \\ 3,29 \\ 3,27 \\ 3,28 \\ 3,31 \\ 3,01 \\ 3,28 \\ 9 \\ 3,28 \\ 3,31 \\ 3,01 \\ 3,28 \\ 3,31 \\ 3,01 \\ 3,28 \\ 3,31 \\ 3,01 \\ 3,28 \\ 3,31 \\ 3,01 \\ 3,28 \\ 3,31 \\ 3,01 \\ 3,28 \\ 3,31 \\ 3,01 \\ 3,28 \\ 3,31 \\ 3,01 \\ 3,28 \\ 3,31 \\ 3,01 \\ $	$\begin{array}{c} 21.85 & 9\\ 39.30 & \\ 39.30 & \\ 7.15 & \\ 5.99 & \\ 5.29 & \\ 4.482 & \\ 4.482 & \\ 4.04 & \\ 4.04 & \\ 4.04 & \\ 4.24 & \\ 4.24 & \\ 4.24 & \\ 4.24 & \\ 3.33 & \\ 3.55 & \\$	$\begin{array}{c} 37.11 \\ 939.33 \\ 349.33 \\ 14.74 \\ 9.69 \\ 5.82 \\ 5.12 \\ 4.65 \\ 4.32 \\ 4.65 \\ 4.32 \\ 4.65 \\ 4.32 \\ 4.65 \\ 3.73 \\ 3.34 \\ 4.07 \\ 3.34 \\ 3.24 \\ 4.07 \\ 3.34 \\ 3.24 \\ 3.34 \\ 3.22 \\ 3.13 \\ 3.41 \\ 3.34 \\ 3.22 \\ 2.31 \\ 3.41 \\ 3.34 \\ 2.82 \\ 2.97 \\ 2.94 \\ 2.92 \\ 2.97 \\ 2.94 \\ 2.92 \\ 2.92 \\ 2.97 \\ 2.94 \\ 2.92 \\ 2.97 \\ 2.94 \\ 2.92 \\ 2.9$	$\begin{array}{c} 48.22 & 9\\ 39.36 & \\ 9.07 & \\ 9.07 & \\ 4.99 & \\ 4.53 & \\ 4.20 & \\ 3.95 & \\ 3.$	8 56.66 (2) 14.54 8.98 6.76 6.76 8.98 6.76 8.98 6.76 4.43 3.35 3.29 3.20 3.20 3.21 3.33 3.22 3.22 3.22 3.22 3.22 2.96 2.96 2.87 2.81 2.87 2.81 2.73 2.41 2.65 2.63 2.41	9 63.28 9 8.30 6.68 8.90 6.68 8.90 6.68 3.32 4.36 4.36 3.59 3.44 3.31 3.12 3.31 3.31 3.31 3.31 3.31 3.31 3.31 2.98 2.88	10 668.63 9 39.40 14.42 5.46 4.30 3.92 2.57 2.73 2.73 2.77 2.73 2.77 2.73 2.77 2.73 2.57 2.57 2.57 2.51 2.59 2.57 2.51 2.59 2.57 2.51 2.59 2.57 2.51 2.57 2.57 2.57 2.57 2.51 2.51 2.51 2.57 2.57 2.57 2.57 2.57 2.57 2.57 2.57 2.57 2.51 2.51 2.57 2.57 2.57 2.51 2.51 2.51 2.57 2.57 2.57 2.51 2.	$\begin{array}{c} 39.42\\ 39.42\\ 39.43\\ 4.5\\ 6.52\\ 5.37\\ 4.67\\ 3.87\\ 3.63\\ 3.63\\ 3.63\\ 3.63\\ 2.96\\ 2.82\\ 2.72\\ 2.64\\ 2.55\\ 2.5$	$\begin{array}{c} 84.87 & 9\\ 39.43 \\ 14.25 \\ 8.64 \\ 5.27 \\ 4.57 \\ 4.57 \\ 3.35 \\ 2.33 \\ 3.18 \\ 5.27 \\ 4.57 \\ 4.57 \\ 2.53 \\ 2.95 \\ 2.57 \\ 2.53 \\ 2.57 \\ 2.44 \\ 2.32 \\ 2.41 \\ 2.34 \\ 2.34 \\ 2.34 \\ 2.34 \\ 2.31 \\ 2.3$	$\begin{array}{c} 93.10 \\ 9 \\ 39.45 \\ 8.56 \\ 6.33 \\ 5.17 \\ 4.47 \\ 3.67 \\ 2.95 \\ 2.51 \\ 2.84 \\ 2.76 \\ 2.84 \\ 2.76 \\ 2.84 \\ 2.76 \\ 2.84 \\ 2.76 \\ 2.84 \\ 2.76 \\ 2.84 \\ 2.84 \\ 2.76 \\ 2.84 \\ 2.28 \\ 2.21 \\ 2.20 \\ 2.23 \\ 2.21 \\ 2.20 \\ 2.22 \\ 2.21 \\ 2.20 \\ 2.21 \\ 2.21 \\ 2.21 \\ 2.22 \\ 2.22 \\ 2.22 \\ 2.22 \\ 2.21 \\ 2.22 \\ 2.22 \\ 2.22 \\ 2.21 \\ 2.22 \\ 2.22 \\ 2.22 \\ 2.21 \\ 2.22 \\ 2.22 \\ 2.22 \\ 2.22 \\ 2.21 \\ 2.22 \\ 2.2$	$\begin{array}{c} 97.25 \\ 139.46 \\ 8.51 \\ 6.28 \\ 5.12 \\ 4.42 \\ 2.83 \\ 3.95 \\ 3.61 \\ 3.95 \\ 3.61 \\ 3.95 \\ 3.61 \\ 2.50 \\ 2.50 \\ 2.41 \\ 2.33 \\ 2.50 \\ 2.41 \\ 2.33 \\ 2.50 \\ 2.41 \\ 2.22 \\ 2.14 \\ 2.22 \\ 2.14 \\ 2.14 \\ 1.88 \\ 1.76 \end{array}$	$\begin{array}{c} 001.4 \\ 39.46 \\ 6.23 \\ 5.07 \\ 4.36 \\ 6.23 \\ 5.07 \\ 4.36 \\ 3.51 \\ 3.89 \\ 3.53 \\ 3.89 \\ 3.33 \\ 3.89 \\ 2.73 \\ 2.64 \\ 2.50 \\ 2.44 \\ 2.51 \\ 2.50 \\ 2.44 \\ 2.51 \\ $	$\begin{array}{c} 005.6 \\ 139.47 \\ 39.47 \\ 14.04 \\ 8.41 \\ 5.01 \\ 4.31 \\ 3.51 \\ 4.33 \\ 3.54 \\ 3.34 \\ 3.34 \\ 3.34 \\ 2.91 \\ 2.58 \\ 2.51 \\ 2.58 \\ 2.51 \\ 2.44 \\ 2.33 \\ 2.29 \\ 2.55 \\ 2.31 \\ 2.44 \\ 2.33 \\ 2.29 \\ 2.55 \\ 2.41 \\ 2.32 \\ 2.25 \\ 2.41 \\ 2.32 \\ 2.25 \\ 2.41 \\ 2.32 \\ 2.25 \\ 2.31 \\ 2.41 \\ 2.32 \\ 2.25 \\ 2.31 \\ 2.25 \\ 2.32 \\ 2.25 \\ 2.31 \\ 2.41 \\ 1.51$	$\begin{array}{c} 009.8 \\ 13.994\\ 8.36\\ 6.12\\ 4.96\\ 4.25\\ 3.78\\ 3.45\\ 3.78\\ 3.45\\ 3.78\\ 3.20\\ 2.85\\ 2.27\\ 2.22\\ 2.28\\ 2.38\\ 2.32\\ 2.27\\ 2.22\\ 2.38\\ 2.38\\ 2.32\\ 2.18\\ 2.11\\ 2.08\\ 2.05\\ 2.03\\ 2.00\\ 1.98\\ 1.94\\ 1.84\\ 1.94\\ 1.84\\ 1.94\\ 1.53\\ 3.52\\ 1.98\\ 1.96\\ 1.53\\ 1.98\\ 1$	$\begin{array}{c} 014.0 \ 1\\ 39.49 \\ 13.95 \\ 8.31 \\ 6.07 \\ 4.90 \\ 4.20 \\ 3.73 \\ 3.39 \\ 3.14 \\ 2.94 \\ 2.55 \\ 2.46 \\ 2.38 \\ 2.32 \\ 2.20 \\ 2.16 \\ 2.11 \\ 2.08 \\ 2.04 \\ 2.01 \\ 1.98 \\ \end{array}$	$\begin{array}{c} 018.3\\ 39.50\\ 13.90\\ 8.26\\ 6.02\\ 4.85\\ 4.14\\ 3.67\\ 3.308\\ 2.72\\ 2.30\\ 2.49\\ 2.40\\ 2.32\\ 2.25\\ 2.19\\ 2.25\\ 2.19\\ 2.09\\ 2.04\\ 2.00\\ 1.97\\ 1.91\\ 1.91\\ \end{array}$		P
v ₁ =2 v ₂ =100 (range 60-120) F=3.93,3.80	2 1 3 1 4 1 5 6 7 8 9 9 10 11 12 13 14 15 16 16 17 13 14 15 16 17 12 13 14 15 16 16 17 18 19 20 20 20 20 20 20 20 20 20 20	$\begin{array}{c} 17.79 & 79\\ 88.51 & 30\\ 77.44 & 1\\ 12.22 & 1\\ 10.01 & \\ 8.81 & \\ 8.87 & \\ 7.57 & \\ 7.57 & \\ 7.57 & \\ 7.57 & \\ 7.57 & \\ 7.57 & \\ 7.57 & \\ 7.57 & \\ 7.57 & \\ 6.94 & \\ 6.94 & \\ 6.12 & \\ 6.04 & \\ 6.41 & \\ 6.30 & \\ 6.20 & \\ 6.41 & \\ 6.30 & \\ 6.20 & \\ 5.98 & \\ 5.98 & \\ 5.87 & \\ 5.83 & \\ 5.75 & \\ 5.75 & \\ 5.75 & \\ 5.75 & \\ 5.75 & \\ 5.69 & \\ 5.59 & \\ 5.57 & \\ 5.5$	$\begin{array}{c} 9.50 & 8.4\\ 9.50 & 8.4\\ 9.900 & 5.7\\ 8.43 & 7.26 \\ 6.54 & 6.54 \\ 5.100 & 5.71 \\ 4.86 \\ 4.51 & 4.76 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.497 \\ 4.22 \\ 4.22 \\ 4.20 \\ 3.80 \\ 3.80 \\ 3.80 \\ \end{array}$	$\begin{array}{c} 4.16\ 8\\ 39.16\ 4\\ 39.16\ 4\\ 15.48\\ 15.48\\ 7.76\\ 6.60\\ 5.89\\ 4.83\\ 4.47\\ 4.54$	$\begin{array}{c} 99.58 \\ 9.39.25 \\ 9.39.25 \\ 9.60 \\ 9.60 \\ 9.60 \\ 6.23 \\ 5.525 \\ 5.55 \\ 5$	$\begin{array}{c} 21.85 & 9\\ 39.30 & \\ 39.30 & \\ 7.15 & \\ 5.99 & \\ 5.29 & \\ 4.82 & \\ 4.84 & \\ 4.94 & \\ 4.94 & \\ 4.94 & \\ 4.94 & \\ 4.94 & \\ 3.89 & \\ 3.33 & \\ 3.58 & \\ 3$	$\begin{array}{c} 37.11 \\ 939.33 \\ 349.37 \\ 14.74 \\ 9.69 \\ 5.82 \\ 5.12 \\ 4.65 \\ 4.32 \\ 4.65 \\ 4.32 \\ 4.65 \\ 3.73 \\ 3.34 \\ 4.07 \\ 3.38 \\ 3.73 \\ 3.34 \\ 3.22 \\ 3.13 \\ 3.41 \\ 3.34 \\ 3.28 \\ 3.41 \\ 3.34 \\ 3.28 \\ 2.97 \\ 2.94 \\ 2.92 \\ 2.9$	48.22 9 339.36 9.07 6.85 5.70 4.53 4.53 4.53 4.53 3.61 3.36 3.36 3.36 3.36 3.36 3.36 3.01 2.93 2.93 2.93 2.93 2.97 2.87 2.87 2.87 2.88 2.75 2.75 2.75 2.75 2.75 2.75 2.75 2.75	8 56.66 9 39.37 4.54 4.54 4.54 4.54 4.43 3.65 3.25 3.25 3.25 3.20 3.22 3.061 3.23 3.20 3.22 3.01 3.22 3.00 3.22 9.31 2.91 2.81 2.81 2.85 2.75 2.65	9 63.88 9 8.90 6.68 5.52 4.36 4.82 4.36 5.52 2.84 4.33 3.78 3.78 3.59 2.93 2.84 2.93 2.93 2.93 2.93 2.70 2.70 2.70 2.57	10 68,63 9 39,40 14,42 5,46 6,62 5,46 4,30 3,96 3,72 3,25 3,25 3,25 2,87 2,67 2,67 2,67 2,67 2,53 2,67 2,53 2,51 2,54 2,55 2,5	$\begin{array}{c} 776.71 \\ 339.42 \\ 14.34 \\ 8.75 \\ 6.52 \\ 5.37 \\ 4.20 \\ 3.62 \\ 3.63 \\ 3.28 \\ 3.28 \\ 3.28 \\ 3.28 \\ 3.28 \\ 3.28 \\ 3.28 \\ 3.05 \\ 2.96 \\ 2.89 \\ 2.82 \\ 2.77 \\ 2.72 \\ 2.64 \\ 2.60 \\ 2.57 \\ 2.54 \\ 2.54 \\ 2.54 \\ 2.54 \\ 2.54 \\ 2.54 \\ 2.54 \\ 2.41 \\ 2.49 \\ 2.47 \\ 2.41 \\ 2.41 \\ \end{array}$	84.87 9 39.43 14.25 6.43 5.277 4.10 3.33 3.377 3.52 2.86 2.85 2.85 2.577 2.62 2.577 2.53 2.547 2.547 2.547 2.34 2.34 2.34 2.32 2.31 2.34 2.32 2.31 2.34 2.32 2.31	$\begin{array}{c} 93.10 \\ 9\\ 39.45 \\ 14.17 \\ 8.56 \\ 6.33 \\ 5.17 \\ 4.00 \\ 3.67 \\ 3.47 \\ 3.07 \\ 2.76 \\ 2.56 \\ 2.51 \\ 2.56 \\ 2.51 \\ 2.42 \\ 2.39 \\ 2.36 \\ 2.33 \\ 2.23 \\ 2.33 \\ 2.21 \\ 2.20 \end{array}$	$\begin{array}{c} 97.25 \\ 1\\ 33.46 \\ 14.12 \\ 8.51 \\ 6.28 \\ 3.95 \\ 5.12 \\ 4.42 \\ 3.95 \\ 3.61 \\ 3.37 \\ 3.61 \\ 3.33 \\ 3.61 \\ 3.33 \\ 2.89 \\ 2.79 \\ 2.70 \\ 2.70 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.41 \\ 2.22 \\ 2.42 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.24 \\ 2.22 \\ 2.24 \\ 2.2$	$\begin{array}{c} 001.4 \\ 39.46 \\ 14.08 \\ 8.46 \\ 6.23 \\ 5.07 \\ 4.36 \\ 3.39 \\ 3.56 \\ 3.31 \\ 3.12 \\ 2.984 \\ 2.64 \\ 2.57 \\ 2.57 \\ 2.57 \\ 2.57 \\ 2.57 \\ 2.39 \\ 2.35 \\ 2.31 \\ 2.35 \\ 2.31 \\ 2.13 \\ 2.11 \\ 2.113 \\ 2.11$	$\begin{array}{c} 005.6 \\ 39.47 \\ 14.04 \\ 8.41 \\ 6.18 \\ 3.51 \\ 3.26 \\ 3.26 \\ 3.26 \\ 2.91 \\ 2.78 \\ 2.67 \\ 2.58 \\ 2.51 \\ 2.48 \\ 2.33 \\ 2.29 \\ 2.25 \\ 2.21 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.20 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.25 \\ 2.21 \\ 2.25 \\ 2.25 \\ 2.21 \\ 2.25 \\$	$\begin{array}{c} 009.8 \\ 13.994\\ 8.36\\ 6.12\\ 4.96\\ 4.25\\ 3.78\\ 3.45\\ 3.78\\ 3.45\\ 3.78\\ 3.20\\ 2.85\\ 2.27\\ 2.22\\ 2.28\\ 2.38\\ 2.32\\ 2.27\\ 2.22\\ 2.38\\ 2.38\\ 2.37\\ 2.20\\ 1.98\\ 1.94\\ 1.94\\ 1.94\\ 1.53\\ 3.55\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.94\\ 1.53\\ 1.95\\ 1.53\\ 1.95\\ 1.53\\ 1.95\\ 1.53\\ 1.95\\ 1.53\\ 1.95\\ 1.53\\ 1.95\\ 1.53\\ 1.95\\ 1.53\\ 1.95\\ 1$	$\begin{array}{c} 0.14 & 0 & 1\\ 39.49 & \\ 8.31 & \\ 6.07 & \\ 4.90 & \\ 4.90 & \\ 4.90 & \\ 3.73 & \\ 3.39 & \\ 2.73 & \\ 2.73 & \\ 2.75 & \\ 2.46 & \\ 2.32 & \\ 2.25 & \\ 2.46 & \\ 2.32 & \\ 2.25 & \\ 2.46 & \\ 2.32 & \\ 2.20 & \\ 2.11 & \\ 2.25 & \\ 2.46 & \\ 2.01 & \\ 1.91 & \\$	018.3 39.50 13.26 6.02 4.14 4.85 4.14 4.85 4.13 3.33 3.67 3.367 2.88 2.73 2.60 2.40 2.240 2.240 2.240 2.240 2.240 2.19 2.13 2.09 2.10 3.187 1.97 1.91 1.91 1.91 1.91 1.91 1.91 1.9	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	P





Probability intermediate intermediate between 0.05 and 0.025 (actually, 0.0323)

v ₁ =2	TABLE 5 (Contd.) P(F) = 0.05	D(E)-0.05
	f_2 f_1 1 2 3 4 5 6 7 8 9 10 12 15 20 24 30 40 60 120 ∞	P(F)=0.05
v ₂ =100	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
(60-120)	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	
	$ \begin{array}{ccccccccccccccccccccccccc$	
F=3.15,3.07	$ \begin{array}{ccccccccccccccccccccccccc$	
at P(F)=0.05	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
F=3.93,3.80	23 4.23 3.37 2.98 2.74 2.59 2.47 2.39 2.32 2.27 2.22 2.15 2.07 1.99 1.95 1.90 1.85 1.80 1.75 1.69 27 4.21 3.35 2.96 2.77 2.57 2.267 2.213 2.06 1.97 1.93 1.88 1.84 1.79 1.73 1.67 28 4.20 3.34 2.95 2.71 2.56 2.46 2.37 2.39 2.24 2.10 2.104 1.90 1.85 1.80 1.75 1.69 298 4.20 3.34 2.95 2.77 2.55 2.43 2.35 2.18 2.10 2.146 1.91 1.87 1.82 1.77 1.65 298 1.83 3.32 2.92 2.69 2.53 2.43 2.35 2.18 2.10 2.04 1.99 1.84 1.79 1.74 1.66 30 4.17 3.32 <t< td=""><td></td></t<>	
at P(F)=0.025	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	
E -2.55	P(F) = 0.025	P(F)=0.025
F _{xspec} =3.55	f_1 1 2 3 4 5 6 7 8 9 10 12 15 20 24 30 40 60 120 ∞	
п	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
ĮĻ	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	
•	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	
Prob interm.	$ \begin{array}{ccccccccccccccccccccccccc$	
between 0.05	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	
and 0.025	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	
(actually,	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	
0.0323)	120 5.1 3.80 .23 2.89 2.67 2.52 2.59 2.60 2.12 2.16 1.16 1.83 1.71 1.64 1.57 1.48 1.39 1.27 1.00	

P(F)=0.025